Practice: Find the derivative

① $f(x) = (x^5 + 7e^x)\sqrt{x}$

② $g(p) = \cos(3p^2 - \pi)$

③ $B(t) = 3^{t^2}$

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① $f'(x) = (5x^4 + 7e^x)\sqrt{x} + (x^5 + 7e^x)(x^{1/2})'$

$= (5x^4 + 7e^x)\sqrt{x} + (x^5 + 7e^x)\frac{1}{2} \cdot x^{-1/2}$

$= (5x^4 + 7e^x)\sqrt{x} + \frac{(x^5 + 7e^x)}{2\sqrt{x}}$

② $g'(p) = -\sin(3p^2 - \pi) \cdot (3p^2 - \pi)'$

$= -\sin(3p^2 - \pi) \cdot 6p$

③ $B'(t) = \left[(e^{\ln 3})^{t^2}\right]' = \left[e^{(\ln 3)t^2}\right]'$

$= e^{(\ln 3)t^2} \cdot \left[(\ln 3)t^2\right]'$

$= e^{(\ln 3)t^2} \cdot 2(\ln 3)t = \boxed{2(\ln 3)t \cdot 3^{t^2}}$

Exponential Rule for derivatives

$$(a^x)' = (\ln(a))a^x$$ for any positive # a.

Proof: $(a^x)' = \left[\left(e^{(\ln a)}\right)^x\right]' = \left(e^{\ln(a)\cdot x}\right)' = e^{\ln(a)x}\cdot \ln(a)$

$= \ln(a)\cdot e^{\ln(a)\,x} = \boxed{\ln(a)\cdot a^x}$. $\square$.

An application of derivatives: projectile motion.

Physics:

s = position (a lot of times – vertical position)

v = velocity (often vertical velocity)

a = acceleration:

V = instantaneous rate of change of position $= \frac{ds}{dt} = s'$ time.

a = instantaneous rate of change of velocity $= \frac{dv}{dt} = \frac{d}{dt}\left(\frac{ds}{dt}\right)$

$= \frac{d^2 s}{dt^2} = s'' = v'$

(second derivative of s)

Example: If $s = 3.5t^2 - 12t + 20$, where t is measured in seconds and s is measured in feet, find the velocity and acceleration.

$$v = s' = \boxed{7t - 12 \left(\frac{\text{feet}}{\text{sec}}\right)}$$

$\frac{ds}{dt}$ ← feet, ← seconds

$$a = s'' = (7t-12)' = \boxed{7\frac{\text{ft}}{\text{sec}^2}}$$

$\frac{dv}{dt}$ ← feet/sec, ← sec

acceleration due to gravity at or near the surface of the earth is a constant

$$a = -9.8\, m/s^2 = -32\, ft/s^2$$

vertical acceleration — objects in "free fall" — no external forces.

Example Suppose Austin throws a baseball with initial velocity 75 miles/hr; at an angle of $30°$ from the horizontal, starting at a height of 5.0 ft.

1. When does the ball reach its maximum height?
2. What is the maximum height?
3. When does the ball hit the ground?
4. How far did the ball go?

Horizontal motion (without external forces) is constant velocity, zero acceleration.

Let's think about this:

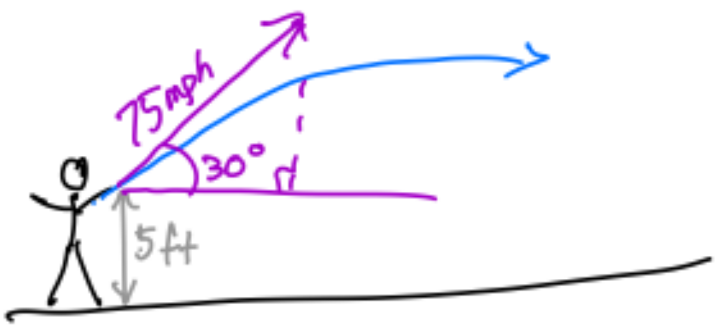
vertical position $s(t) = ??$

$$s''(t) = a(t) = -32 \text{ ft/sec}^2$$

constant

$$\Rightarrow s'(t) = v(t) = \underline{-32t + C}$$

the only fcn so that $v'(t) = -32$.



$$75\text{mph} \cdot \sin(30°) = 37.5 \frac{\text{miles}}{\text{hour}} = 37.5 \frac{5280 \text{ ft}}{3600 \text{ s}}$$
$$= 55 \text{ ft/s}$$

$$75 \text{ mph} \cos(30°) = 75 \text{ mph} \cdot \frac{\sqrt{3}}{2} = \frac{75\sqrt{3}}{2} \frac{\text{miles}}{\text{hour}} = \left(\frac{75\sqrt{3}}{2}\right) \frac{5280 \text{ ft}}{3600 \text{ s}}$$
$$= 95.3 \text{ ft/s}$$

When $t = 0$

$v(t) = 55 \text{ ft/s}$

$$v(t) = -32\underset{0}{t} + C = 55$$

$$\Rightarrow C = 55$$

vertical velocity.

$$\boxed{v(t) = -32t + 55}$$

(a) at maximum height $v(t) = 0$

$$-32t + 55 = 0$$

$$55 = 32t \qquad t = \frac{55}{32} \text{ s}$$

$$\Rightarrow \boxed{t = 1.71 \text{ s}}$$